HW2004, Set 1 solution

```
> restart;
```

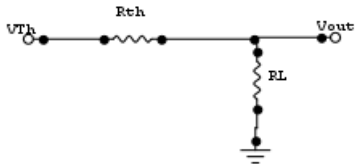
```
> i:=V_in/(R1+R2);
```

$$i := \frac{V\_in}{R1 + R2}$$

```
> V_out:=i*R2;
```

$$V\_out := \frac{V\_in\ R2}{R1 + R2}$$

We know from class how to construct a Thevinen's equivalent circuit for this voltage divider circuit, so we can simplify the circuit to:



where

```
> V_th:=V_in*R2/(R1+R2);
```

$$V\_th := \frac{V\_in\ R2}{R1 + R2}$$

and

```
> R_th:=R1*R2/(R1+R2);
```

$$R\_th := \frac{R1\ R2}{R1 + R2}$$

Adding RL to the output forms a voltage divider between R_th and RL, so for the loaded circuit,

```
> V_out:=V_th*RL/(R_th+RL);
```

$$V\_out := \frac{V\_in\ R2\ RL}{(R1 + R2)\left(\frac{R1\ R2}{R1 + R2} + RL\right)}$$

In the no-load condition, V_out is just V_th, so we are looking for the RL that gives V_out = V_th/2:

```
> solve(V_out=V_th/2,RL);
```

$$\frac{R1\ R2}{R1 + R2}$$

This just just the voltage divider output impedance, R_th.

```
>
```

Now we need to find the RL that draws maximum power PL. First we need to express PL in terms of the circuit parameters. Vout is the voltage across RL, and let's call IL the current through RL. Then

```
> PL:=IL*V_out;
```

$$PL := \frac{IL\ V\_in\ R2\ RL}{(R1 + R2)\left(\frac{R1\ R2}{R1 + R2} + RL\right)}$$

```
> IL:=V_out/RL;
```

$$IL := \frac{V\_in\ R2}{(R1 + R2)\left(\frac{R1\ R2}{R1 + R2} + RL\right)}$$

So the power is

```
> PL;
```

$$\frac{V\_in^2\ R2^2\ RL}{(R1 + R2)^2\left(\frac{R1\ R2}{R1 + R2} + RL\right)^2}$$

```
> simplify(%);
```

$$\frac{RL\ R2^2\ V\_in^2}{(R1\ R2 + RL\ R1 + RL\ R2)^2}$$

We want to find the RL that draws max power, so differentiate PL with respect to RL and set it equal to zero.

```
> equation1:=   0=diff(PL,RL);
```

$$equation1 := 0 = -\frac{2\ V\_in^2\ R2^2\ RL}{(R1 + R2)^2\left(\frac{R1\ R2}{R1 + R2} + RL\right)^3} + \frac{V\_in^2\ R2^2}{(R1 + R2)^2\left(\frac{R1\ R2}{R1 + R2} + RL\right)^2}$$

```
> solve(equation1,RL);
```

$$\frac{R1\ R2}{R1 + R2}$$

This is just R_th, the output impedance of the original voltage divider. This is an important general result. You draw
the maximum power from a circuit when the load matches the output impedance.

```
>
```